GEOMETRY.

497. Proposed by NATHAN ALTSHILLER, University of Oklahoma.

Find the locus of the mid-point of the segment determined by two spheres on any line passing through any point common to the two spheres.

498. Proposed by FRANK R. MORRIS, Glendale, California.

To trisect an angle ABC, on BA and BC take D and E equidistant from B. Using DE as a diameter draw the semicircle $DFGE$. With the same radius and D and E as centers draw arcs locating the points F and G on this semicircle. Connect F and G with B. Prove that this method trisects a right angle and a straight angle and that it does not trisect an oblique angle.

CALCULUS.

415. Proposed by GEORGE PAASWELL, New York City, N. Y.

If r is the distance from a fixed point (x, y, z) to a variable point (x', y'), in the plane $z = 0$; determine the values of the integrals $\int\int r\,dx'dy'$ and $\int\int \log(z + r)dx'dy'$ for the two cases

(a) when the integration is extended over the surface of the circle of radius R; and

(b) when the integration is extended over the surface of the rectangle of dimensions a, b.

These integrals are special cases of the direct and logarithmic potentials, the densities of the surface distributions being taken as unity.

416. Proposed by CHARLES N. SCHMALL, New York City, N. Y.

If A be a point on a cycloid and C the corresponding position of the center of the generating circle, show that AC envelops another cycloid half the size of the first.

MECHANICS.

332. Proposed by E. E. MOOTS, University of Arizona.

In any quadrilateral $ABCD$ whose diagonals AB and BD intersect in E, lay off on AC from C, CF equal to AE. Join F to B. Join G, the middle point of BE, to D. On GD lay off GM equal to one-third of GD. Prove that M is the center of gravity of the quadrilateral.

333. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

A flywheel 21 feet in diameter makes 100 revolutions per minute. The weight of a cubic foot of its material is 448 pounds. Show that the intensity of stress on a transverse section of rim, assuming that it is unaffected by the arms, is 1,176 lbs. per sq. in. If the safe stress permissible in the material is 6,000 lbs. per sq. in., show that the greatest speed at which the wheel can be run with safety is about 225 revolutions per minute.

NUMBER THEORY.

251. Proposed by HERMAN ROLAND KATNICK, Chicago, Ill.

Determine the character of the positive integer n so that the Diophantine system

$$z + n = x^2, \qquad z - n = y^2$$

shall have an integral solution; and exhibit a method for finding all the values of x, y, z for a given n of such character.

252. Proposed by E. J. MOULTON, Northwestern University.

(A) Show that the number of integers x on the interval $10^r \leqq x < 10^{r+1}$ which do not contain the digit 1 at least p times, $p \leqq r$, is

$$9\cdot\{[\text{first } p \text{ terms of expansion of } (9 + 1)^r] - {}_rC_{p-1}\cdot 9^{r-p}\}$$

where ${}_rC_{p-1}$ is the coefficient of x^{p-1} in the expansion of $(1 + x)^r$.

(B) Show that the number of integers x on the interval $10^r \leqq x < 10^{r+1}$ which do not contain the digit 0 at least p times, $p \leqq r$, is

$$9\cdot[\text{first } p \text{ terms of expansion of } (9 + 1)^r].$$